Math 10A with Professor Stankova
Quiz 5; Wednesday, 9/27/2017
Section #106; Time: 10 AM
GSI name: Roy Zhao **Name:** ____________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False It is possible that repeatedly using Newton's method brings you further and further from the root.
2. **TRUE** False The Taylor series for $x^4 + 3x^2 - 5x + 1$ is $x^4 + 3x^2 - 5x + 1$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) Approximate $\sqrt[3]{8.12}$ using second order Taylor series. You may leave your answer as a sum of fractions.

Solution: We want to approximate the function $f(x) = \sqrt[3]{x}$. A good base point is a value nearby which is $\sqrt[3]{8} = 2$. So expanding $f(x)$ around $x = 8$ gives

$$f(x) \approx f(8) + f'(8)(x-8) + \frac{f''(8)}{2}(x-8)^2 = 2 + \frac{x-8}{3 \cdot 2^2} - \frac{(x-8)^2}{9 \cdot 2^5}$$

$$= 2 + \frac{x-8}{12} - \frac{(x-8)^2}{288}.$$

Now we plug in $x = 8.12$ to get

$$\sqrt[3]{8.12} \approx 2 + \frac{0.12}{12} - \frac{(0.12)^2}{288} = \boxed{2 + \frac{1}{100} - \frac{1}{2 \cdot 10^4}}.$$

(b) (1 point) When using Newton's method to find a zero of a function $f(x)$, what is the formula for the next guess x_1 if my current guess is x_0?

Solution:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

(c) (4 points) Use Newton's method once to approximate $\sqrt[3]{8.12}$.

Solution: Our function that we want to find a zero of is not $\sqrt[3]{x}$ but $x^3 - 8.12$. Our initial guess is $x_0 = 2$, not $x = 8$ because we want to guess the final answer. Now we use the above formula to get that our next guess is

$$x_1 = 2 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{8 - 8.12}{3 \cdot 2^2} = 2 + \frac{0.12}{12} = \boxed{2.01}.$$